technical monograph 42

Understanding Decibels (dB or not dB)

Floyd D. Jury

Technical Consultant
Fisher Controls International, Inc.

This document is designed to provide the learner with a working knowledge of scientific notation, logarithms, dB (decibels), sound pressure level, dB gain, and the various relationships between these factors.

FISHER®

UNDERSTANDING DECIBELS
(dB or not dB)

INTRODUCTION

Many fields of engineering and science are so intertwined with the mathematical concepts of logarithms and decibels that it is vitally important for one to thoroughly understand their meaning and their operation. The fields of acoustics and process control are two important examples. Since decibels are not a common part of most people's daily lives, this document is devoted to the development of a thorough understanding of decibels as they are commonly used.

EXPONENTIAL RELATIONSHIPS

Most of us are quite familiar with the real number system and what it means. We use these real numbers to express a quantity of something. This quantity is usually referenced to some absolute reference value, normally zero. For example, if I don't work, I don't earn any money; i.e., for zero days work, I receive zero dollars. On the other hand, if my rate of pay is 10 dollars per day, then I know that after the first day I will have 10 dollars, and 100 dollars after 10 days. This type of relationship is called linear.

Not everything that we encounter in real life can be described by this type of linear relationship. For example, suppose that I bet $1 on a 10 to 1 shot at the racetrack. If my horse comes in, my winnings on the first race would be $10. Since my luck seems to be good, I might continue to bet my winnings from each race at the same 10 to 1 odds. If I won the first four races, my winnings for each race would be;

first race	$10
second race	$100
third race	$1000
fourth race	$10,000

It's obvious that this relationship is far from linear. The word we frequently use to describe this type of relationship is, "exponential." We might further describe this relationship by saying that the winnings increase by a factor of ten, or by a power of ten, on each race. This simply means that we are describing the increase in my winnings by an exponent of ten, hence the name, "exponential relationship." A table will help to describe this relationship:

RACE	WINNINGS	WINNINGS USING EXPONENTS OF 10	EXPONENT
1	$10	$\$10^1$	1
2	$100	$\$10^2$	2
3	$1000	$\$10^3$	3
4	$10,000	$\$10^4$	4

Even though the quantity of my winnings increases in an exponential manner, it is interesting to note that the exponent of ten increases in a linear manner.

A useful way to look at this exponential relationship is to think of the exponent as being the number of times that we multiply one (or unity) by the factor of ten. Thus, 10^2 really means;

$$10^2 = 1x10x10 = 100$$

Using this example as a guide, we would expect to write the quantity 10^4 as;

$$10^4 = 1x10x10x10x10 = 10{,}000$$

This brings up an interesting point of speculation. Suppose that we elect to multiply the one by no tens at all! How would we express this in exponential notation and what would be the result? If we simply follow the pattern of the previous examples, we can write this as;

$$1x(\text{no tens}) = 10^0 = 1$$

Very interesting! Now, let's push the issue even further. Instead of **multiplying** the one by ten, suppose we elect to **divide** by ten! If we assume that we can extend our exponential notation to cover this situation, we would interpret 10^{-1} as;

$$10^{-1} = 1/(\text{one ten}) = 1/10 = 0.1$$

Following the clue provided by this example, we can now use this exponential notation to express 0.001 as;

$$0.001 = 1/1000 = 1/(10x10x10) = 1/10^3 = 10^{-3}$$

Just to make certain that we have it all straight in our minds, lets summarize what we have done so far in a table.

$$\begin{aligned} 10^4 &= 10{,}000 \\ 10^3 &= 1{,}000 \\ 10^2 &= 100 \\ 10^1 &= 10 \\ 10^0 &= 1 \\ 10^{-1} &= 0.1 \\ 10^{-2} &= 0.01 \\ 10^{-3} &= 0.001 \\ 10^{-4} &= 0.0001 \end{aligned}$$

We all know that numbers don't always come in nice neat factors of ten. We frequently get numbers like 5327, 16.52, or 0.00139. In cases like this, it is helpful to remember that we can manipulate these numbers into other forms with the help of multiplication or division. For example;

5327	= 5.327x1000	= 5.327×10^{3}
16.52	= 1.652x10	= 1.652×10^{1}
0.00139	= 1.39/1000	= 1.39×10^{-3}

This type of exponential representation of numbers is known as scientific notation. You may have noticed that we wrote each of these cases so that the lead number was always between one and ten. If we make a standard practice of always doing that on this type of notation, we refer to it as "standard scientific notation."

This type of exponential relationship, based on the number 10, is quite common in the scientific world. For example, in early studies of speech, hearing, and sound, researchers at Bell Labs found this common base number of ten very useful. In rating the magnitudes of various sounds, they found it necessary to establish a base or reference level of sound. The reference point chosen was that level of sound which could just barely be detected by an average young man with normal hearing. This "threshold of hearing," as it was called, was statistically determined using many, many tests.

Using the threshold of hearing as a reference, these researchers proceeded to measure other sounds common to the environment, ranging from soft whispers to speeding trains. The range of sound powers that were encountered was tremendous. Furthermore, they discovered that, within this range, the apparent loudness did not increase in a linear fashion with sound power, but rather increased quite rapidly by factors of ten.

The researchers found themselves dealing with everyday sounds that ranged from a million times as powerful as the threshold of hearing sound, or perhaps only one tenth as powerful. They found it very cumbersome to express these various sound power levels this way, so they decided to use the exponential representation. Even so, it was still cumbersome to say that a sound was 10^{6} more powerful than the threshold of hearing, or conversely 10^{-4} as powerful as the hearing threshold.

To simplify things even further, some of the researchers adopted their own shorthand notation and simply said that the POWER was "up 6" or "down 4." What they were stating was simply the exponent on the common base number, ten. "Up 6" meant that the sound was 10^{6} times more powerful than the threshold of hearing; whereas, "down 4" meant that the sound was only 10^{-4} as powerful.

DEFINING dB

To help clarify this new notation even further, the Bell Labs researchers coined a new measurement unit which they called a "BEL" in honor of Alexander Graham Bell, and added it to their exponential notation. Now, if a sound was 1000 times as powerful as the threshold sound, they could say that it was, "up 3 BELS."

It became immediately apparent that they were going to need a unit of sound measurement that was somewhat smaller than the BEL. They divided the BEL into ten parts, called "deciBELS" since "deci" means "one-tenth." Of course, these Bell Labs researchers knew all along that the exponents they had been using were really called, "logarithms."

In order to see this more clearly, we can refresh our memory on the definition of logarithms; i.e.,

$$10^{L} = N \qquad (1)$$

where:

10 = Base
L = Logarithm (or Log)
N = Number (or anti-Log)

Therefore, using this definition, the base ten logarithm of the number 100 is equal to 2; i.e., $10^{2} = 100$.

The logarithm definition above is often written in another equivalent form.

(Logarithm definition)

$$L = \mathrm{Log}_{10} N \qquad \text{or} \qquad 10^{L} = N \qquad (2)$$

This is read as, "L equals the base ten logarithm of the number N." As was mentioned before, the base ten is the most common base number that is used in our physical systems. For this reason, the logarithm to the base ten is called a "common logarithm." When referring to the common logarithm, it is sometimes customary to omit the base ten designation and write the common logarithm simply as;

$$L = \mathrm{Log}\, N$$

Here the symbol, "Log" denotes a "common" logarithm. Using the definitions given, we can develop a small table of logarithms as follows:

NUMBER (N)	LOGARITHM (L)
0.01	-2
0.1	-1
1	0
10	1
100	2

You have probably already noticed that the common logarithm (L) is the same exponent of the common base ten that was used in the previous sound measurements. In that case, the number (N) was the ratio of the measured sound power to the threshold sound power; i.e., the logarithm (L) is identical to what we previously defined as a BEL.

$$\text{BEL} = \text{Log}_{10} N$$

$$\text{BEL} = \text{Log (Measured Power)/(Reference Power)} \qquad (3)$$

and, since the deciBEL is really only one-tenth of a BEL we can write

(Number of deciBELS) = dB = 10x(Number of BELS)

therefore,

$$\text{dB} = 10\text{Log (Measured Power)/(Reference Power)} \qquad (4)$$

From this equation, we see that the term "deciBEL," or more commonly, "decibel," can be abbreviated to the simple notation, "dB." The following table will confirm our understanding of this relationship.

POWER RATIO (Meas. Power)/(Ref. Power)	DECIBELS (dB)
0.01	-20 dB
0.1	-10 dB
1	0 dB
10	10 dB
100	20 dB

So far, we have only talked about measurements of the POWER level. It is frequently desirable to take measurements of POTENTIAL. In a pneumatic or an acoustic system, pressure represents the system potential. In an electrical system, the variable that represents system potential is voltage (E). If we were to review our high school physics book, we would recall the electrical formulas;

$$E = IR$$
$$\text{Power} = EI$$

We can manipulate these two equations into another form which is very enlightening; i.e.,

$$\text{Power} = E^2/R \qquad (5)$$

Since R represents a system constant and E represents the potential, we can generalize this equation by saying, "Power is proportional (α) to the square of the potential." In view of this relationship, we can express the power ratio that we developed previously as;

$$\text{(Meas.Pwr/Ref.Pwr)} \ \alpha \ \text{(Meas.Pot./Ref.Pot.)}^2$$

We can now express the dB equation as a function of the potential as well as the power:

$$\text{dB} = 10\text{Log (Measured Power/Reference Power)}$$

$$\text{dB} = 10\text{Log (Measured Potential/Reference Potential)}^2$$

This second equation should remind us of a theorem that we learned in high school math; i.e.,

$$\text{Log } N^2 = 2\text{Log } N$$ (See Appendix for a proof)

If we apply this theorem to the second equation of dB immediately above, we can now express the dB definition in both of its useful forms;

dB = 10Log (Meas. Power/Ref. Power) (6)

and

dB = 20Log (Meas. Potential/Ref. Potential) (7)

These two important equations may appear to be different, but they really express the SAME definition in two equivalent ways.

EXAMPLE: Assume an electrical circuit with a resistance of 50 ohms. If we apply 100 volts to this circuit, equation (5) above shows that the amount of power that would be dissipated would be 200 watts. If we were to change the voltage on this circuit to 200 volts, the amount of power dissipated would now be 800 watts. This represents a change of +6 dB. We doubled the voltage and quadrupled the power. Either way, it calculates out to +6 dB. As we indicated earlier, pressure is the potential in an acoustic system. Thus, if we doubled the acoustic pressure we would quadruple the acoustic power, and the change would be +6 dB.

PROCESS CONTROL APPLICATION

The concept of measuring things in decibels is also used extensively in the field of automatic process control, so a review of this application is also appropriate.

People who work with process control systems find the concept of GAIN very useful. In very simple terms, gain is essentially the amount of change in output we get from a device or system for a given change in input; i.e.,

$$\text{GAIN} = \frac{\text{(Magnitude of output change)}}{\text{(Magnitude of input change)}} \qquad (8)$$

Even though gain appears to be a type of ratio, it is not a true dimensionless ratio. Many control devices have units of output that are different from the units of input measurement. A simple pneumatic positioner-actuator device is a good example. In a typical situation, this device might require a 12 psi change in the input bellows pressure to produce a 2 inch change in stem position. In this example, the gain would be

GAIN = 2 in./12 psi = 0.167 in./psi

Before we even begin to consider applying the concept of decibels to this gain, we need to get rid of these units of measurement and form a true dimensionless ratio. Control people handle this problem in two ways. We'll investigate both.

Nearly all control devices that we are interested in have a normal operating range, or rated span, on both their input and output. The positioner-actuator referenced above has rated input and output spans of 12 psi and 2 inches respectively.

It is possible to express gain in a slightly different way so that we can achieve the necessary dimensionless ratio; i.e., we normalize the gain by expressing the input and output change as a percent of the rated span.

$$\text{NORMALIZED GAIN (N.G.)} = \frac{\%\text{ change in output}}{\%\text{ change in input}} \qquad (9)$$

Normalized Gain (N.G.) is sometimes referred to as magnitude ratio, but what we call it is not as important as knowing how to use it. We can calculate the normalized gain for the positioner-actuator example above as follows.

N.G. = 100%/100% = 1.0

Let's assume now that some type of internal adjustment is made on the positioner-actuator mechanism so that the same12 psi change in bellows pressure only produces 1.0 inch of stem motion. If we make a new gain calculation, we get

GAIN = 1 in./12 psi = 0.083 in./psi

This adjustment has cut the gain in half as we can easily see from the normalized gain calculation.

N.G. = 50%/100% = 0.5

Remember, the rated output span is still 2.0 inches.

Next, we might ask what the ratio of the new gain is to the original gain. We can look at this problem in two different ways; e.g., we can think about the ratio of the normalized gains, or the ratio of the actual gains.

RATIO = 50%/100% = 0.5
Or (10)
RATIO = (0.083 in./psi)/(0.167 in./psi) = 0.5

When we are talking about a gain change, it makes little difference whether we use the ratio of the actual gains or the ratio of the normalized gains; the result is the same.

Now comes a loaded question! How many dB does this gain change represent? You may already have guessed that the correct answer is –6 dB, but let's find out why.

When dealing with gain in decibels (dB), the only two equations that we have to chose from are equations (6) and (7). When the control people originally decided that they needed a dB relationship to express gain, they had to decide which of these two equations were the most appropriate.

From a theoretical point of view, there was no valid reason to choose either one, because the gain isn't necessarily either a power or a potential ratio. On the other hand, there is nothing so sacred about the dB definition that we have to restrict its use to either the power or potential ratio. Actually, the dB definition is simply a logarithmic function that can be applied to any dimensionless ratio, provided that the result has some useful meaning to us.

Strictly from an intuitive point of view, the control people decided that the concept of gain seemed more nearly related to the potential ratio than to the power ratio. Hopefully, you will think so too! If not, you're stuck with it anyway. The decision was made some time ago and that's the way it's always done now! (NOTE: This decision was made easier by the fact that some of the early control theory work was done on electronic systems where the gain of the amplifier is the ratio of the output voltage over the input voltage.)

Thus, the definition of gain in dB is

GAIN (dB) = 20Log (Gain Ratio) (11)

We still refer to the result as gain, but we give it units of dB.

The Gain Ratio in equation (11) can actually take several forms. As we saw in equations (10) that this ratio can be either the ratio of actual gains or the ratio of normalized gains.

In addition, we can develop a Gain Ratio when we change a system gain from one value to another; i.e.,

Gain Ratio = (New gain)/(Initial gain)

For example, if we change the gain of a system or device from a gain = 4 to a new gain = 2, we would have a gain ratio = 0.5. We would say that this represents a gain change of –6 dB, or that we have cut the gain by 6 dB.

QUESTION: If the normalized gain ratio is 1.0, what would be the gain in units of dB?

ANSWER: ZERO! This particular value has great significance to control people.

FREQUENCY RESPONSE

Control people have found that under dynamic conditions, the gain of most control devices will vary as a function of the frequency of the input signal. For example, let's refer to the original positioner-actuator example above where the unit was adjusted so that a 12 psi change in the input would result in a 2 inch change in stem position.

If we were to place the stem position in mid-range and then vary the input pressure very slowly in a sinusoidal manner with a total amplitude of 12 psi (± 6 psi), the device would have plenty of time to respond and we would expect a sinusoidal output variation of 2 inches total amplitude (± 1 inch).

Under these quasi-static conditions, we would have to say that the gain (normalized) of the device was 1.0 (i.e., 0 dB).

On the other hand, if we began varying the input pressure at faster and faster frequencies, but still at a total amplitude of 12 psi, we can visualize that the device just wouldn't have time to respond to these faster changes, and the stem would no longer be able to make its full 2 inches of sinusoidal travel. In other words, the gain of the device has decreased simply because the frequency of the input signal changed.

Likewise, at certain frequencies we might even encounter some resonant conditions where we might get more output motion than we expected and the gain increases. Any way you cut it, the gain varies as a function of frequency due to the dynamic characteristics of the device.

It is common and useful in the control business to make a plot of how the gain of a device changes with frequency. This is known as a "frequency response plot," or alternatively as a "Bode plot." For reasons of convenience, these plots are usually made in terms of dB Gain versus Frequency.

SUMMARY

For ease of reference, we will review some of the fundamentals here. For example, the two equivalent forms of the definition of the common (base 10) logarithm are

(Logarithm definition)

$$L = \mathrm{Log}_{10} N \qquad \text{or} \qquad 10^{L} = N \tag{2}$$

The two equivalent forms of the definition of dB (decibels) are

$$\mathbf{dB = 10Log\ (Meas.\ Power/Ref.\ Power)} \tag{6}$$

and

$$\mathbf{dB = 20Log\ (Meas.\ Potential/Ref.\ Potential)} \tag{7}$$

NOTE: In air-based acoustic systems, the reference values are

Reference Potential = $2\text{x}10^{-5}$ Pascals

Reference Power = 10^{-12} Watts

The definition of dB Gain is

$$\mathbf{GAIN\ (dB) = 20Log\ (Gain\ Ratio)} \tag{11}$$

Finally, there are two fundamental principles that are worth remembering:

± 6 dB represents a factor of 2 change in potential or gain, whereas ±3 dB represents a factor of 2 change in power.

A given dB change represents the same physical change in the environment regardless of whether it is calculated from the power or the potential ratio.

THE END

APPENDIX

****

PROOF THAT LOG N^2 = 2LOG N

We can verify this by starting with N^2 and the original definition of logarithm; i.e.,

$L = \text{Log } N$
$10^L = N$

$N^2 = (N)(N) = (10^L)(10^L) = 10^{2L}$

applying the logarithm definition to this statement gives,

$2L = \text{Log } N^2$
$2(\text{Log } N) = \text{Log } N^2$
$2\text{Log } N = \text{Log } N^2$

****

SPECIAL LOG OPERATIONS

There are two special operations involving logarithms which are often used. These two relationships are

$\text{Log } AB = \text{Log } A + \text{Log } B$

and

$\text{Log } A/B = \text{Log } A - \text{Log } B$

The first of these operations can be proven quite easily from the definition of the logarithm; i.e., let

$10^{La} = A$ and $10^{Lb} = B$

which corresponds to the equivalent

$La = \text{Log}_{10} A$ and $Lb = \text{Log}_{10} B$

Now, from the above

$AB = (10^{La})(10^{Lb}) = 10^{La+Lb}$

Thus, from the general logarithm definition

$\text{Log}_{10} AB = La + Lb = \text{Log}_{10} A + \text{Log}_{10} B$

Likewise, the reader should be able to use the same procedure to prove

$\text{Log } A/B = \text{Log } A - \text{Log } B$

****

Fisher Controls International, Inc.
205 South Center Street
Marshalltown, Iowa 50158
Phone: (641) 754-3011
Fax: (641) 754-2830
Email: fc-valve@frmail.frco.com
Website: www.fisher.com

FISHER®

D350674X012 / Printed in the U.S.A./1-99

FISHER-ROSEMOUNT™ Managing The Process Better.™